KTU · ECE · ELECTROMAGNETICS

Smith Chart & Waveguides

Module V comprehensive notes — graphical analysis of transmission lines, impedance matching, waveguide modes and field solutions.

SECTION 01

The Smith Chart

The Smith Chart is a graphical tool for analysing transmission lines. It gives a visual representation of how the impedance of a transmission line and its reflection coefficient change as we move along the line — without requiring complex calculations.

The chart consists of a circular plot with families of interlaced circles. Its main advantage is that transmission line characteristics can be determined without complex computational work.

KEY IDEA

Normalised Impedance

Since every TL has a different Z_0, we normalise all impedances: $z_L = Z_L / Z_0$. This means a single Smith Chart works for any transmission line.

REFLECTION COEFFICIENT

Γ_L Formula

The chart is built from $\Gamma_L = (Z_L - Z_0)/(Z_L + Z_0)$, which in normalised form becomes $\Gamma_L = (z_L - 1)/(z_L + 1)$.

BOUNDARY POINTS

Short & Open Circuits

Left extreme: r=0, x=0 → *short circuit.*

Right extreme: r=∞, x=∞ → *open circuit.*

Reflection Coefficient on the Chart

Both z_L and Γ_L are complex. Writing $z_L = r + jx$ and $\Gamma_L = u + jv$, we get:

DERIVATION

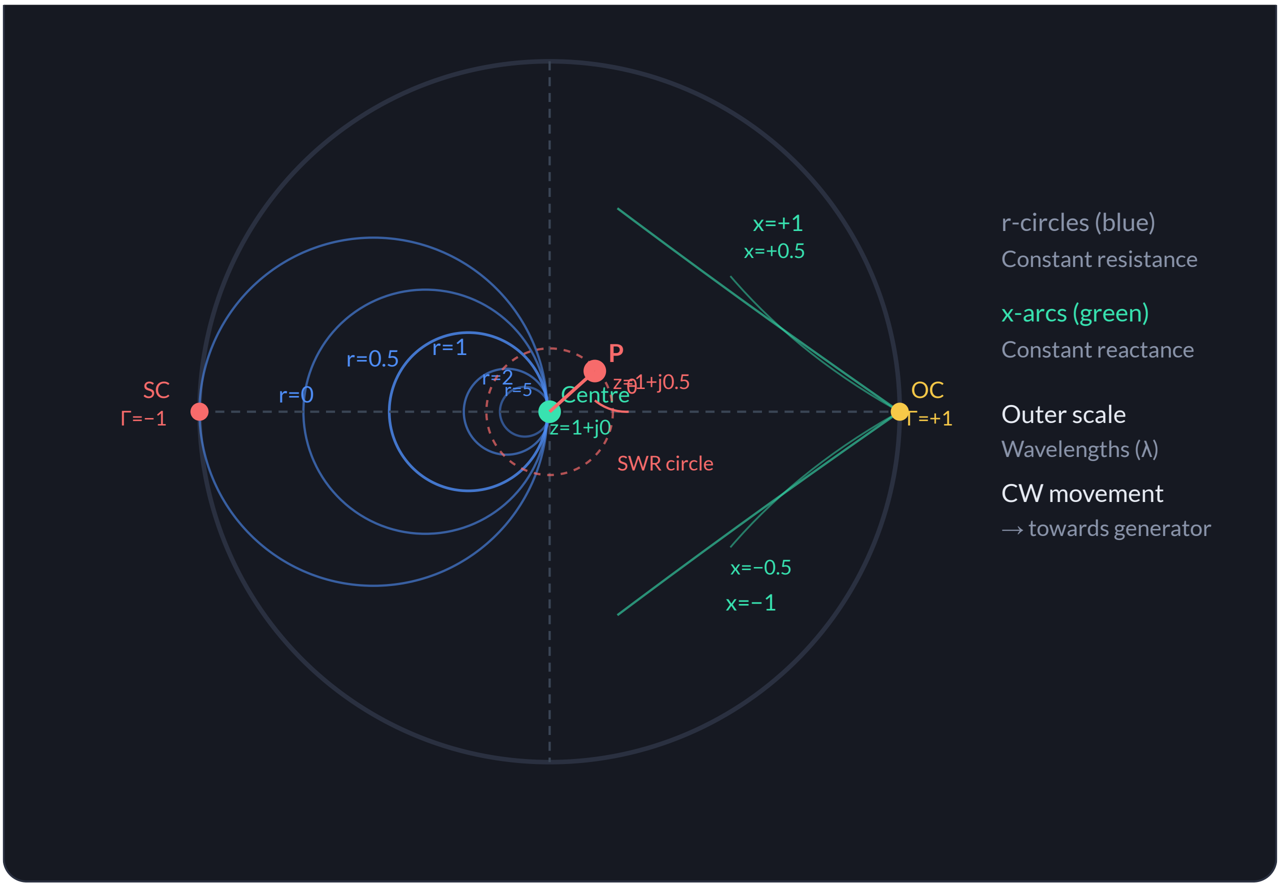
$$z_L = (1 + \Gamma_L) / (1 - \Gamma_L)$$

$$r = (1 - u^2 - v^2) / (1-u)^2 + v^2$$

$$x = 2v / (1-u)^2 + v^2$$

The Smith Chart lies within the unit circle: $|\Gamma_L| \le 1$. The reflection coefficient is written as $\Gamma_L = |\Gamma_L|\, e^{j\theta} = \Gamma_r + j\Gamma_i$.

SMITH CHART — CONCEPTUAL LAYOUT



Important Points about the Smith Chart

POINT 1

Revolution = λ/2

A complete 360° revolution around the chart equals a distance of λ/2 on the transmission line. Clockwise = towards generator. Counter-clockwise = towards load.

POINT 2

Three Scales

Outermost: distance from generator (in λ). Middle: distance from load (in λ). Innermost: angle protractor for measuring θ of Γ.

POINT 3

Dual Use

The Smith Chart can serve as both an impedance chart and an admittance chart. The r and x circles correspond to g and b circles when used for admittance.

SECTION 02

Construction — r and x Circles

Resistance Circles (r-circles)

By manipulating the real part of $z_L = (1+\Gamma)/(1-\Gamma)$, we arrive at the equation of a circle:

R-CIRCLE EQUATION

$$(u - r/(1+r))^2 + v^2 = (1/(1+r))^2$$

Centre: (r/(1+r), 0) Radius: 1/(1+r)

NORMALISED R	RADIUS	CENTRE (U, V)
0	1	(0, 0)

0.5	2/3	(1/3, 0)
1	1/2	(1/2, 0)
2	1/3	(2/3, 0)
5	1/6	(5/6, 0)
∞	0	(1, 0)

All r-circles pass through the point (1,0) on the right of the chart. As r→∞ the circles collapse to that single point.

Reactance Circles (x-circles)

Similarly, from the imaginary part:

X-CIRCLE EQUATION

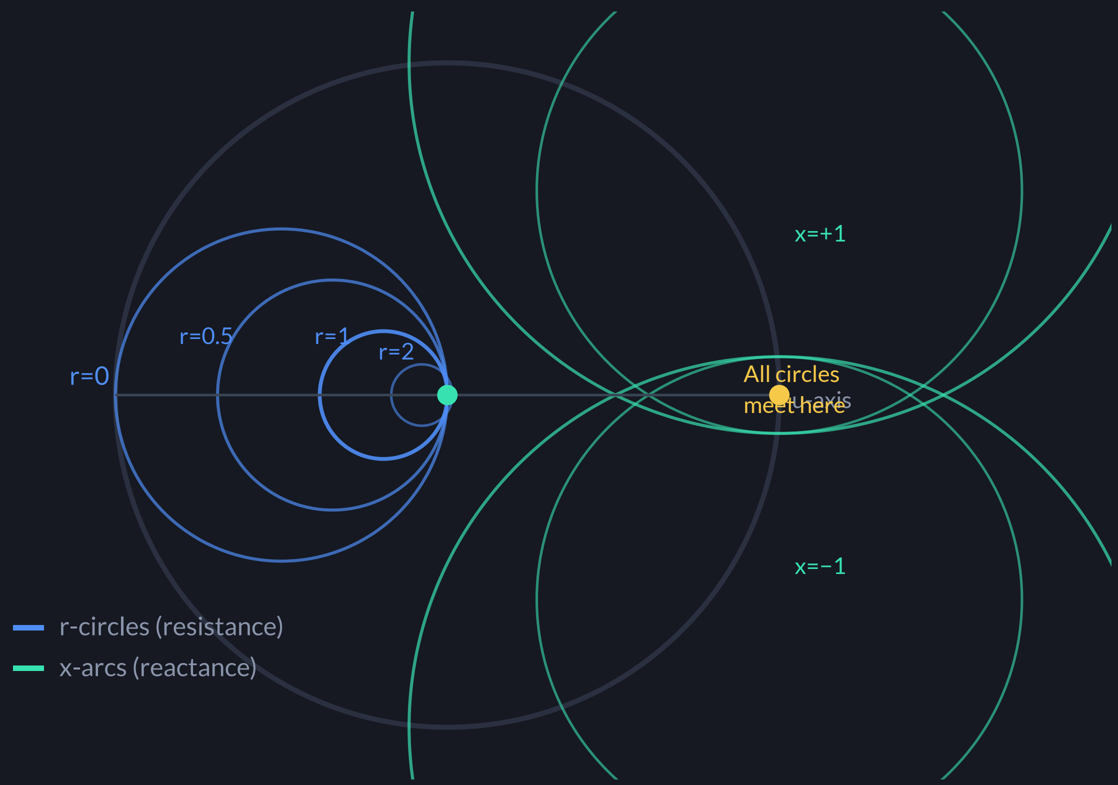
$$(u - 1)^2 + (v - 1/x)^2 = (1/x)^2$$

Centre: (1, 1/x) Radius: |1/x|

NORMALISED X	RADIUS	CENTRE
0	∞	(1, ∞) — horizontal axis
±0.5	2	(1, ±2)
±1	1	(1, ±1)
±2	1/2	(1, ±1/2)
±5	1/5	(1, ±1/5)
±∞	0	(1, 0)

✓ All x-circles also pass through (1,0). Positive x (inductive) arcs are in the upper half; negative x (capacitive) arcs are in the lower half. Superimposing r-circles and x-circles onto the unit circle gives the complete Smith Chart.

R-CIRCLES AND X-ARCS — SUPERIMPOSED



SECTION 03

Applications of the Smith Chart

- → Plotting complex impedance $Z_L = R + jX$ on the chart
- → Finding reflection coefficient Γ_L (magnitude and phase) for a given load
- → Finding the Standing Wave Ratio (SWR) for any impedance
- → Finding the admittance Y_L of a given impedance Z_L
- → Finding the input impedance Z_{in} at any point along a transmission line

→ Matching a transmission line to a load using a single series stub

(a) Plotting Impedance

Normalise the load impedance: $z_L = Z_L/Z_0$. Locate the intersection of the r-circle and x-arc corresponding to the real and imaginary parts.

> i Example: $Z_L = 300 - j25\ \Omega$ on a $50\ \Omega$ line → $z_L = 6 - j0.5$. Since $x = -0.5$, the point Q is in the lower half of the chart at the intersection of the r=6 circle and x=−0.5 arc.

(b) Finding Reflection Coefficient Γ_L

PROCEDURE

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1. Normalise and locate point P on chart
2. Draw line from centre O to P
3. Extend to meet r=0 circle at point G
4. |Γ_L| = OP / OG (OG corresponds to |Γ|=1)
5. θ = angle of OP with horizontal axis
Example: Z_L=60+j40, Z0=50 → z_L=1.2+j0.8 → |Γ|=0.35∠56°
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(c) Standing Wave Ratio (SWR)

Once z_L is located (say at point P), draw a circle centred at O with radius OP. This is the constant SWR circle (also called the constant $|\Gamma|$ circle).

SWR READING

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Locate point S where the SWR circle meets the positive
real axis (Γ_r-axis).
The value of r at S = SWR value
If Z_L=Z0, z_L=1+j0 → |Γ|=0 → SWR=1 (matched line)
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(d) Load Admittance

If z_L is marked at point P, extend line OP through the centre to the diametrically opposite point P'. The value read at P' gives the normalised admittance y_L.

EXAMPLE (FROM NOTES)

$Z_L = 60+j40$, $Z_0 = 50\ \Omega$

$z_L = 1.2+j0.8$ → P on chart

y_L at P' $= 0.575 - j0.38$

$Y_L = Y_0 \cdot y_L = (1/50)(0.575-j0.38) = 0.0115 - j0.0076$ S

(e) Input Impedance Z_{in}

To find Z_{in} at a distance l from the load:

STEP-BY-STEP

1. Normalise: $z_L = Z_L/Z_0$, locate P
2. Convert l to degrees: 360° ↔ λ/2, so l(in λ) × 720° = angle
3. Rotate clockwise by that angle on SWR circle → new point R
4. Read z_{in} at R, then $Z_{in} = Z_0 \cdot z_{in}$

Example: l=0.4λ on Z_0=75Ω line, Z_L=100+j150Ω

$z_L = 1.33+j2$ → rotate 288° CW → $z_{in}=0.29+j0.64$

$Z_{in} = 75(0.29+j0.64) = (21.75 + j48)\ \Omega$

SECTION 04

Transmission Line Sections as Circuit Elements

From the input impedance formula $Z_{in} = Z_0 [(Z_L + jZ_0 \tan \beta l) / (Z_0 + jZ_L \tan \beta l)]$, special cases arise for open and short-circuited lines.

Open-Circuited Line ($Z_L = \infty$)

RESULT

$$Z_{in} = -jZ_0 \cot(\beta l)$$

Pure reactance. Capacitive when $\beta l < \pi/2$; inductive when $\pi/2 < \beta l < \pi$.

Short-Circuited Line ($Z_L = 0$)

RESULT

$$Z_{in} = +jZ_0 \tan(\beta l)$$

Pure reactance. Capacitive when $\pi/2 < \beta l < \pi$; inductive when $\beta l < \pi/2$.

★ Both open- and short-circuited TL sections can act as inductor or capacitor depending on their length. No physical L or C components are needed at microwave frequencies!

Half-Wavelength Section (λ/2)

L = Λ/2 → BL = Π → TAN(Π) = 0

$$Z_{in} = Z_0 \cdot [Z_L/Z_0] = Z_L$$

A half-wavelength section is transparent — the input impedance equals the load impedance. Useful when physical spacing between two ports is needed without altering electrical behaviour.

Quarter-Wavelength Section (λ/4) — Transformer

L = Λ/4 → BL = Π/2 → TAN(Π/2) = ∞

```
Zin = Z₀² / ZL

∴ Z₀ = √(Zin · ZL)
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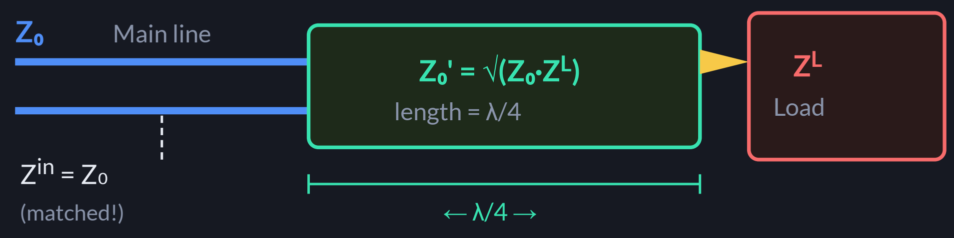
The quarter-wave transformer (also called quarter-wave matching section) is used for impedance matching. To match a load Z_L to a line Z_0, insert a λ/4 section with characteristic impedance $Z_0' = \sqrt{(Z_0 \cdot Z_L)}$.

> ! Example: To match a 120 Ω load to a 75 Ω line, insert a λ/4 section with $Z_0' = \sqrt{(75 \times 120)} = 95\ \Omega$.

QUARTER-WAVE TRANSFORMER — IMPEDANCE MATCHING



SECTION 05

Waveguides

A waveguide is a hollow conducting tube used to transmit EM waves from one point to another. Unlike transmission lines, it can support many field configurations (modes).

TL vs Waveguide Comparison

PROPERTY	TRANSMISSION LINE	WAVEGUIDE

Supported modes	TEM wave only	TE, TM (many modes)
Frequency range	DC to very high f	Only above cutoff f_c
Efficiency at microwave	Low (skin effect, dielectric loss)	High (less loss)
Bandwidth	Narrower	Larger
Filter behaviour	—	High-pass (cannot transmit DC)

! Waveguides act as high-pass filters — they can only transmit frequencies above a minimum cutoff frequency. Below cutoff, the wave is attenuated and does not propagate.

SECTION 06

Modes of Propagation

The waves propagating through a waveguide have infinite possible patterns of electric and magnetic fields — these patterns are called **modes**. Three fundamental mode types exist:

MODE 1

TEM — Transverse Electromagnetic

Both $E_z = 0$ and $H_z = 0$. Both E and H are perpendicular to the direction of propagation. Cannot exist in a hollow waveguide — requires two conductors.

MODE 2

TE — Transverse Electric (H-wave)

$E_z = 0$, $H_z \neq 0$. E is entirely transverse; H has a longitudinal component. Called TE_{mn} mode (m, n = mode indices).

MODE 3

TM — Transverse Magnetic (E-wave)

$H_z = 0$, $E_z \neq 0$. H is entirely transverse; E has a longitudinal component. Called

TM_{mn} mode.

MODE SUMMARY

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TEM: E_z = 0, H_z = 0
TE: E_z = 0, H_z ≠ 0 (dominant in rectangular WG)
TM: H_z = 0, E_z ≠ 0
```

SECTION 07

Parallel Plate Waveguide

Two perfectly conducting parallel plates separated by distance 'a', of infinite extent in y and z directions. The medium between them is lossless dielectric (σ=0) with parameters ε and μ.

Boundary Conditions

BC 1

Tangential E = 0

$E_t = 0$ at the conducting surface. Electric field terminates normally on the conductor.

BC 2

Normal H = 0

$H_n = 0$ at the conducting surface. Magnetic field is tangential to the conductor.

General Field Equations (from Maxwell)

WITH ASSUMPTIONS: PROPAGATION IN Z, UNIFORM IN Y (∂/∂Y = 0)

$$E_x = -(\gamma/h^2) \cdot \partial E_z/\partial x \quad [A]$$

$$H_x = -(\gamma/h^2) \cdot \partial H_z/\partial x \quad [B]$$

$$E_y = (j\omega\mu/h^2) \cdot \partial H_z/\partial x \quad [C]$$

$$H_y = (-j\omega\varepsilon/h^2) \cdot \partial E_z/\partial x \quad [D]$$

where $h^2 = \gamma^2 + \mu\varepsilon\omega^2$

TE Wave Solution (E_z=0)

Solving with BCs E_y=0 at x=0 and x=a gives sinusoidal variation:

TE FIELD COMPONENTS (PARALLEL PLATES)

$$E_x = 0 \qquad E_z = 0$$

$$E_y = A_1 \sin(m\pi x/a)\, e^{-j\beta z}$$

$$H_x = -(\beta/j\mu\omega)\, A_1 \sin(m\pi x/a)\, e^{-j\beta z}$$

$$H_z = -(A_1 m\pi/j\mu\omega a) \cos(m\pi x/a)\, e^{-j\beta z}$$

$h = m\pi/a$, where $m = \pm1, \pm2, \pm3 \ldots (m \neq 0)$

Lowest order mode: TE_{10}

TM Wave Solution (H_z=0)

TM FIELD COMPONENTS (PARALLEL PLATES)

$$E_y = 0 \qquad H_x = 0$$

$$E_x = (\gamma A_4/j\varepsilon\omega) \cos(m\pi x/a)\, e^{-\gamma z}$$

$$E_z = -(hA_4/j\varepsilon\omega) \sin(m\pi x/a)\, e^{-\gamma z}$$

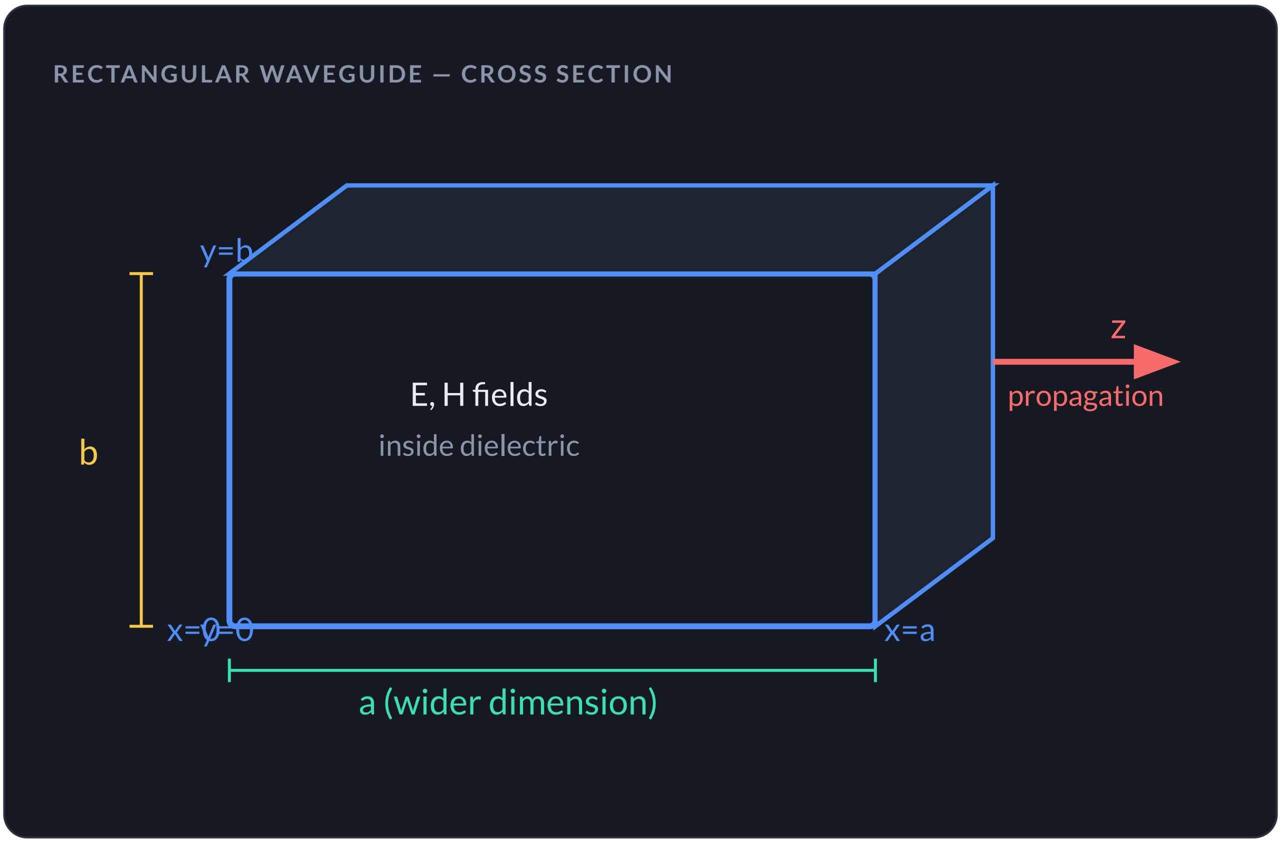
$$H_y = A_4 \cos(m\pi x/a)\, e^{-\gamma z}$$

Lowest order TM mode: TM_0 (m=0 allowed for TM)

SECTION 08

Rectangular Waveguide

A hollow rectangular conducting pipe with cross-section dimensions a (width) and b (height), propagating in the z-direction. More practical than infinite parallel plates.



TE$_{mn}$ Mode in Rectangular Waveguide

For TE modes (E_z=0), solving the wave equation by variable separation gives Hz(x,y,z):

H_Z SOLUTION

$$H_z(x,y,z) = A_{mn} \cos(m\pi x/a) \cos(n\pi y/b) e^{-j\beta z}$$

m = 0,1,2… and n = 0,1,2…, not both zero

ALL TE FIELD COMPONENTS

$$E_x = (j\omega\mu n\pi/h^2 b) A_{mn} \cos(m\pi x/a) \sin(n\pi y/b) e^{-j\beta z}$$

$$E_y = -(j\omega\mu m\pi/h^2 a) A_{mn} \sin(m\pi x/a) \cos(n\pi y/b) e^{-j\beta z}$$

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Hx = (jβmπ/h²a) Amn sin(mπx/a) cos(nπy/b) e^-jβz

Hy = (jβnπ/h²b) Amn cos(mπx/a) sin(nπy/b) e^-jβz
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Cutoff Frequency

CUTOFF FREQUENCY F_C

$$f_c = (v_p/2ab) \sqrt{(mb)^2 + (na)^2}$$

$$= (v_p/2) \sqrt{(m/a)^2 + (n/b)^2}$$

For air-filled WG: $v_p = c = 3\times10^8$ m/s

Cutoff wavelength: $\lambda_c = 2 / \sqrt{(m/a)^2 + (n/b)^2}$

Dominant Mode: TE_{10}

The TE_{10} mode (m=1, n=0) has the lowest cutoff frequency and is the dominant mode in all rectangular waveguides. For TE_{10}:

TE_{10} CUTOFF FREQUENCY

$f_c = v_p/(2a)$ $\quad\lambda_c = 2a$

Phase and Group Velocities

PHASE VELOCITY

$v_p = \omega/\beta$

Velocity of propagation of equiphase surfaces along the guide. Always greater than free-space velocity in a waveguide.

GROUP VELOCITY

$v_g = d\omega/d\beta$

Velocity of energy propagation. Always less than free-space velocity. For lossless media: $v_p \cdot v_g = v_c^2$

KEY RELATIONSHIP

$$v_p \cdot v_g = v_c^2 = c^2$$

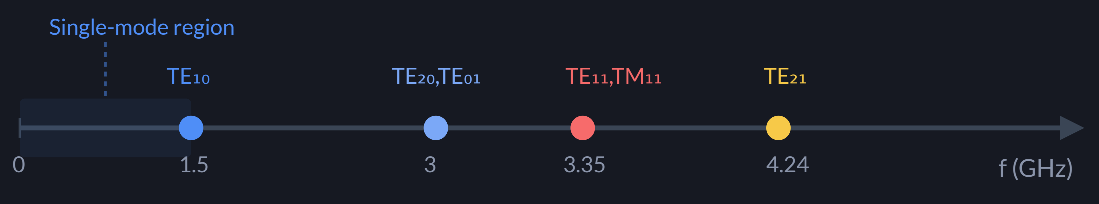
At cutoff: $v_p \to \infty$, $v_g \to 0$ (no energy propagation)

Worked Example — Cutoff Frequencies

Ex Air-filled rectangular WG with a=100mm, b=50mm. Find f_c for TE_{10}, TE_{20}, TE_{01}, TE_{11}.

MODE	M	N	F_C
TE_{10}	1	0	1.5 GHz (dominant)
TE_{20}	2	0	3 GHz
TE_{01}	0	1	3 GHz
TE_{11}	1	1	3.354 GHz
TE_{12}	1	2	6.185 GHz
TE_{21}	2	1	4.243 GHz
TM_{11}	1	1	3.354 GHz (same as TE_{11})

MODE CUTOFF FREQUENCY SPECTRUM (A=100MM, B=50MM)



★ TE_{10} is the dominant mode because it has the lowest cutoff frequency. A waveguide is designed to operate in the TE_{10} mode by choosing the operating frequency to lie between $f_c(TE_{10})$ and $f_c(TE_{20})$. This ensures only the dominant mode propagates — single-mode operation.

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