ELECTROMAGNETIC THEORY

Module IV – Complete Digital Notes

Transmission Lines · Poynting's Theorem · Wave Polarization

- **Transmission Lines** — *Types, parameters, equations*
- **Lossless & Distortionless Lines** — *Special cases & conditions*
- **Wave Propagation** — *Propagation constant, impedance*
- **Reflection & SWR** — *Terminated lines, standing waves*
- **Poynting's Theorem** — *Energy flow in EM fields*
- **Wave Polarization** — *Linear, elliptical, circular*

KTU – B.Tech Electrical & Electronics Engineering

1. TRANSMISSION LINES

Wave propagation can occur in an **unbounded medium** or through **guided structures**. A transmission line (TL) is a classic example of such a guided structure, used in power distribution (low frequency) and communication systems (high frequency). Common examples include twisted pair, coaxial cables, and microstrip lines used in computer networks.

1.1 Types of Transmission Lines

Four standard types of transmission lines are used in practice:

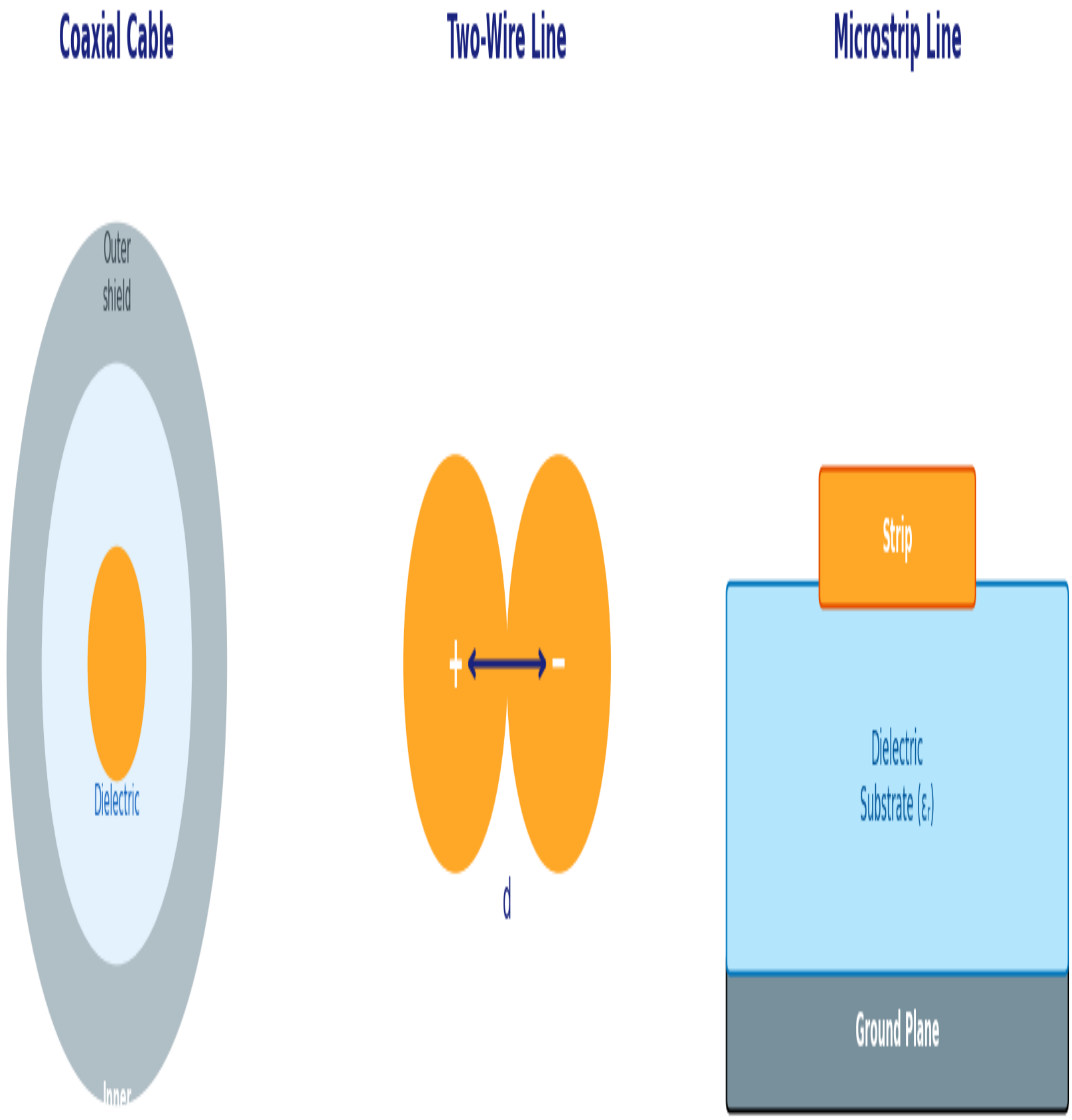

Figure 1.1: Common transmission line types

1.2 Transmission Line Parameters (Distributed)

A transmission line is characterised by **four distributed parameters** (per unit length):

Symbol	Parameter	Unit	Description
R	Resistance/unit length	Ω/m	AC resistance of conductors
L	Inductance/unit length	H/m	External inductance
G	Conductance/unit length	S/m	Dielectric leakage
C	Capacitance/unit length	F/m	Between conductors

Key characteristics of line parameters:

- R, L, G and C are **distributed** along the entire length of the line.
- For each line: **LC =** $\mu\varepsilon$ and **G/C =** σ/ε
- R is the AC resistance per unit length; G is the conductance per unit length.
- L is the *external* inductance per unit length.
- Conductors are characterised by σ_c, μ_c; dielectric by σ, μ, ε.

1.3 Transmission Line Equations

Consider an incremental section of length Δz of a two-conductor TL. The section is modelled using an **L-type equivalent circuit**. Applying Kirchhoff's Voltage and Current Laws yields the TL equations.

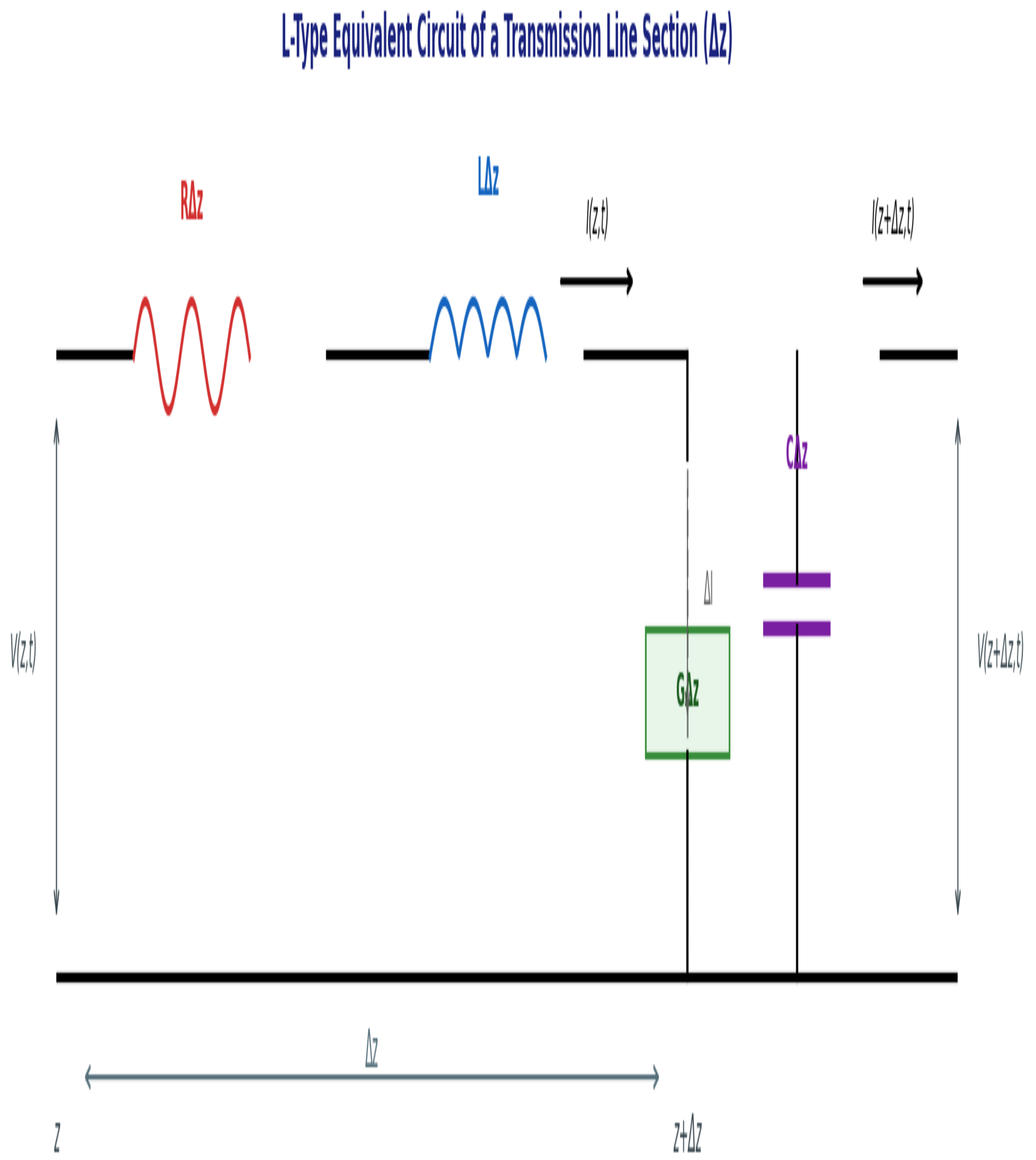

Figure 1.2: L-type equivalent circuit of an incremental TL section

Applying KVL to the outer loop and letting $\Delta z \to 0$:

$$-\partial V(z,t)/\partial z = R \cdot I(z,t) + L \cdot \partial I(z,t)/\partial t \quad \ldots (1)$$

Applying KCL to the main node and letting $\Delta z \to 0$:

$$-\partial I(z,t)/\partial z = G \cdot V(z,t) + C \cdot \partial V(z,t)/\partial t \quad \ldots (2)$$

Assuming **time-harmonic dependence** $V(z,t) = Re[Vs \cdot e^{j\omega t}]$, $I(z,t) = Re[Is \cdot e^{j\omega t}]$ (so $\partial/\partial t \rightarrow j\omega$), the phasor equations become:

$$-dVs/dz = (R + j\omega L) \cdot Is \ \ldots\ (3) \quad -dIs/dz = (G + j\omega C) \cdot Vs \ \ldots\ (4)$$

Taking the double derivative of (3) and substituting (4), the **wave equations** are:

$$d^2Vs/dz^2 - \gamma^2 Vs = 0 \text{ and } d^2Is/dz^2 - \gamma^2 Is = 0$$

Symbol	Name	Unit	Expression
γ	Propagation constant	m■¹	$\gamma = \alpha + j\beta = \sqrt{[(R+j\omega L)(G+j\omega C)]}$
α	Attenuation constant	Np/m	Real part of γ
β	Phase constant	rad/m	Imaginary part of γ

1.4 General Solution of Wave Equations

The general solution of the wave equations consists of two travelling waves:

Vs(z) = V■■e■■■ + V■■e■■ Is(z) = (V■■/Z■)e■■■ – (V■■/Z■)e■■

- The **+z travelling** wave: V■■$e^{-\gamma z}$ (incident wave from generator to load)
- The –**z travelling** wave: V■■$e^{+\gamma z}$ (reflected wave from load to generator)
- **Wavelength**: $\lambda = 2\pi/\beta$
- **Wave velocity**: $u = \omega/\beta = f\lambda$

1.5 Characteristic Impedance (Z■)

Definition: The characteristic impedance Z■ is the ratio of the positively-travelling voltage wave to the corresponding current wave at any point on the line. It is analogous to the intrinsic impedance η of a medium.

Z■ = V■■/I■■ = –V■■/I■■ = $\sqrt{[(R+j\omega L)/(G+j\omega C)]}$

1.6 Special Cases

Case I – Lossless Line (R = 0, G = 0)

Conductors are perfect ($\sigma_c \rightarrow \infty$) and dielectric is lossless ($\sigma=0$). Necessary condition: $R = G = 0$.

$\gamma = j\omega\sqrt{(LC)} \rightarrow \alpha = 0, \beta = \omega\sqrt{(LC)}$ Z■ = $\sqrt{(L/C)}$ (purely real) $u = 1/\sqrt{(LC)}$

Case II – Distortionless Line (R/L = G/C)

A signal normally has many frequency components. In a lossy line, α is frequency-dependent, causing different attenuation of different components — leading to **distortion**. A distortionless line has α independent of frequency and β linearly proportional to frequency.

The general criterion for a distortionless line is: **R/L = G/C**.

$\alpha = \sqrt{(RG)}$ (frequency-independent) $\beta = \omega\sqrt{(LC)}$ (linearly dependent on ω) Z■ = $\sqrt{(L/C)} = \sqrt{(R/G)}$ (real)

Summary: Transmission Line Characteristics

Case	Condition	Propagation Constant γ	Characteristic Impedance Z_0
General (Lossy)	$R \neq 0, G \neq 0$	$\gamma = \sqrt{[(R+j\omega L)(G+j\omega C)]}$	$Z_0 = \sqrt{[(R+j\omega L)/(G+j\omega C)]}$
Lossless	$R=0, G=0$	$\gamma = j\omega\sqrt{(LC)} \rightarrow \alpha=0, \beta=\omega\sqrt{(LC)}$	$Z_0 = \sqrt{(L/C)}$ (real)
Distortionless	$R/L = G/C$	$\gamma = \sqrt{(RG)} + j\omega\sqrt{(LC)}$	$Z_0 = \sqrt{(R/G)} = \sqrt{(L/C)}$ (real)

Figure 1.3: Summary of transmission line characteristics

2. TERMINATED TRANSMISSION LINES

Consider a TL of length l characterised by γ and Z■, connected to load impedance Z_L. Let z = 0 at the load and z = –l at the generator.

2.1 Voltage Reflection Coefficient (Γ_L)

At z = 0, the boundary condition $Z_L = V_s(0)/I_s(0)$ allows solving for V■■ in terms of V■■:

Γ_L = V■■/V■■ = (Z_L – Z■)/(Z_L + Z■)

- Range of Γ_L: –**1** ≤ Γ_L ≤ **1**
- When $\Gamma_L = 0$ (i.e. Z_L = Z■): **matched line** – no reflection
- Current reflection coefficient = $-\Gamma_L$ (opposite sign)
- Voltage and current on the line are superpositions of incident and reflected waves (standing waves)

2.2 Standing Wave Ratio (SWR / VSWR)

When reflections occur, voltage and current vary in magnitude along the line, producing **standing waves** with definite maxima and minima. The VSWR is defined as the ratio of maximum to minimum voltage magnitude:

$$\text{VSWR} = |V_{max}|/|V_{min}| = (1 + |\Gamma_L|)/(1 - |\Gamma_L|)$$

Range of VSWR: **1** ≤ **VSWR** ≤ ∞

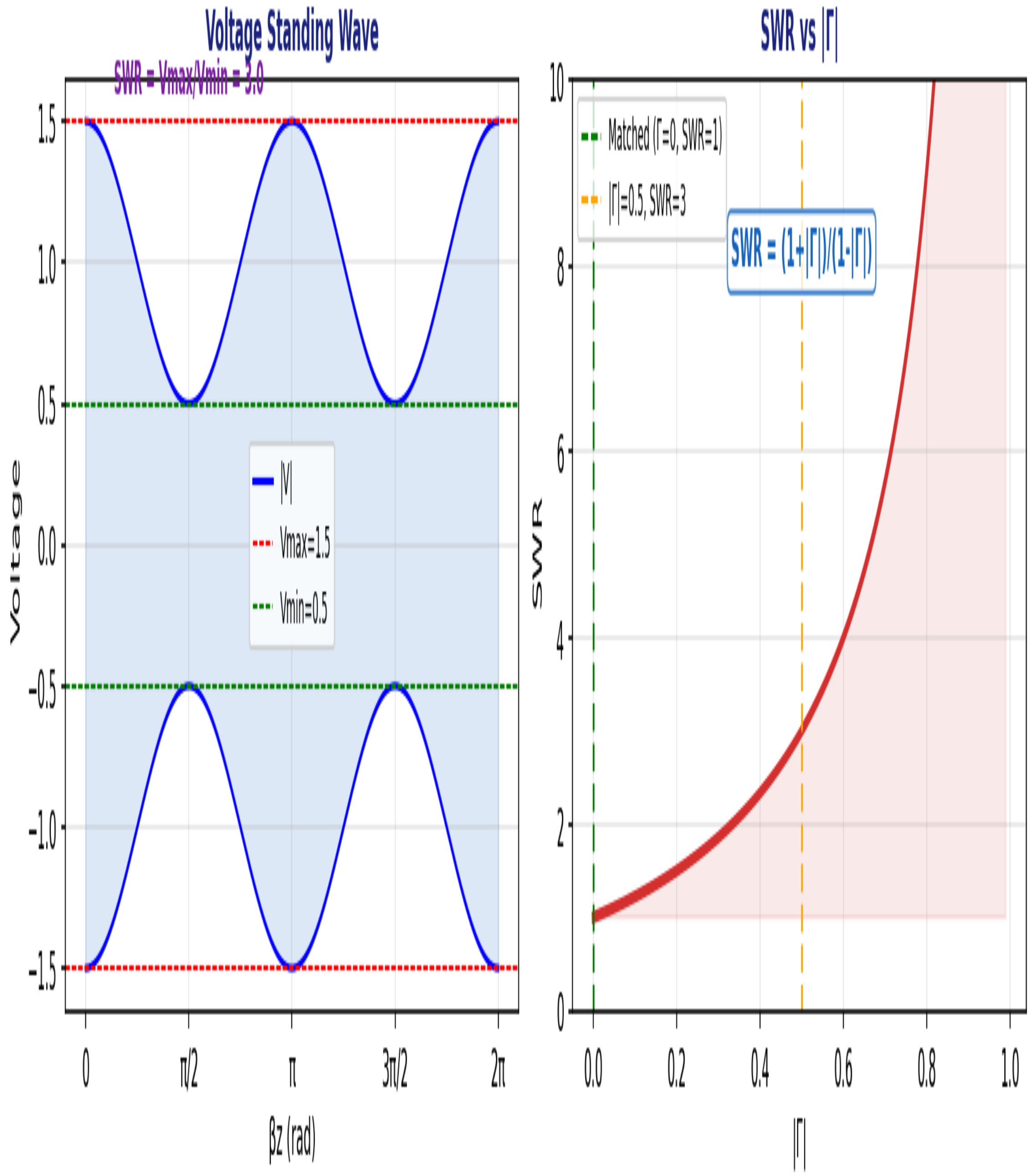

Figure 2.1: Voltage standing wave pattern and SWR vs |Γ|

2.3 Input Impedance (Z_{in})

The input impedance looking into the line at z = –l is:

Z_in = Z■ · [Z_L + Z■·tanh(γl)] / [Z■ + Z_L·tanh(γl)] (general/lossy)

For a **lossless line** (α=0, γ=jβ, tanh(jβl)=j·tan(βl)):

Z_in = Z■ · [Z_L + jZ■·tan(βl)] / [Z■ + jZ_L·tan(βl)]

Special Case	Condition	Z_in	Γ_L	SWR

Shorted Line	Z_L = 0	jZ■·tan(βl)	–1	∞
Open-Circuited Line	Z_L = ∞	–jZ■·cot(βl)	1	∞
Matched Line	Z_L = Z■	Z■	0	1

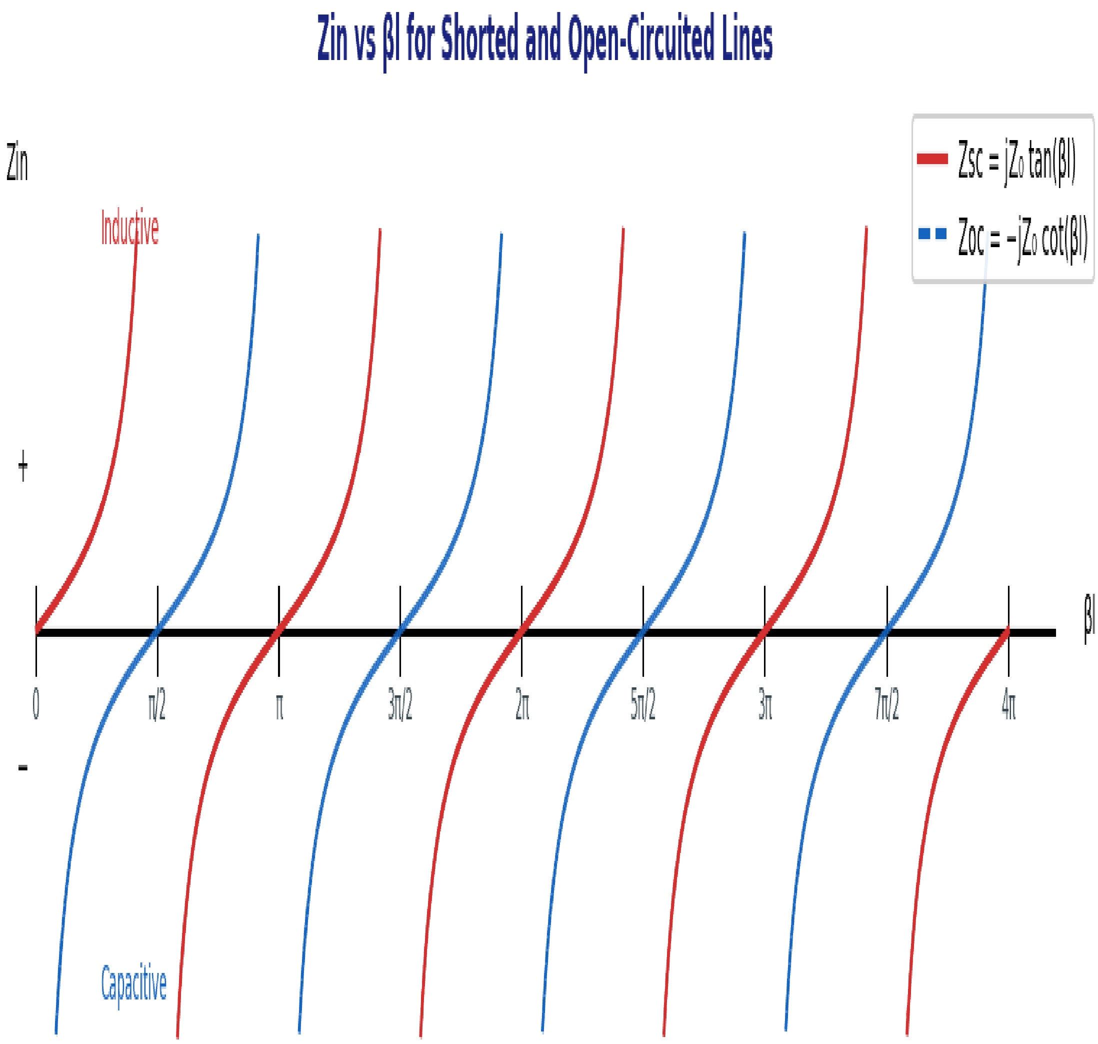

Figure 2.2: Variation of Z_in with βl for shorted and open lines

■ Key Note – Matched Line

When Z_L = Z■ (matched line): Z_in = Z■, Γ_L = 0, and VSWR = 1. This is the most desirable operating condition — the entire wave is transmitted with no reflection. This is the goal in practical RF/microwave systems.

2.4 Average Power on a Lossless Line

The average power at a distance l from the load on a lossless line is:

P_av = (|V■■|²)/(2Z■) · (1 – |Γ_L|²) = P_i – P_r

P_{av} is maximum when $\Gamma_L = 0$ (matched condition). P_i = incident power, P_r = reflected power.

3. POYNTING'S THEOREM

3.1 Poynting Vector

The **Poynting vector** S■ is defined as the vector product of the electric and magnetic field intensities:

S■ = E■ × H■ |S■| = |E■||H■|·sin(E■, H■)

The Poynting vector measures the **rate of flow of energy** of a wave as it propagates. Its direction represents the direction of power flow, and it is perpendicular to the plane containing E and H.

3.2 Poynting's Theorem Statement

Poynting's Theorem

The net power flowing out of a given volume V equals the time rate of decrease of the energy stored within V minus the conduction (ohmic) losses inside V.

■(E■×H■)·dS■ = –d/dt ∫[½εE² + ½μH²]dV – ∫σE²dV

Interpretation of each term:

■(E■×H■)·dS■	Total power leaving the volume (through closed surface S)
d/dt ∫[½εE² + ½μH²]dV	Rate of decrease in stored electric & magnetic energy
∫σE²dV	Ohmic (conduction) power dissipated as heat

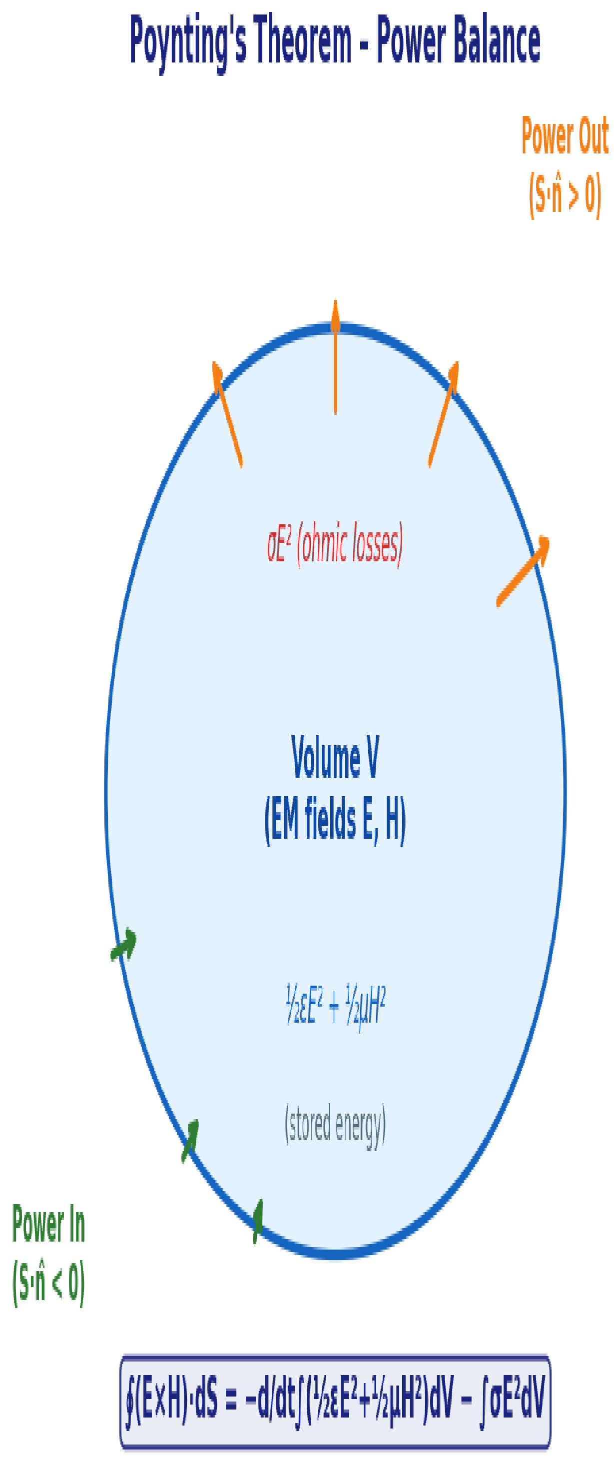

Figure 3.1: Illustration of power balance for EM fields (Poynting's theorem)

Derivation summary: Starting from Maxwell's equations ($\nabla\times E = -\mu\partial H/\partial t$ and $\nabla\times H = \sigma E + \varepsilon\partial E/\partial t$), the divergence of ($E\times H$) is expanded using the vector identity $\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$, and then the divergence theorem is applied to convert the volume integral to a surface integral.

4. POLARIZATION OF PLANE WAVES

The **polarization** of a uniform plane wave is defined as the time-varying behaviour of the electric field vector E■ at some fixed point in space, along the direction of propagation.

For a wave travelling in the +z direction, E■ and H■ lie in the x-y plane. There are **three types** of polarization depending on the amplitudes and phase difference of the x and y components of E■.

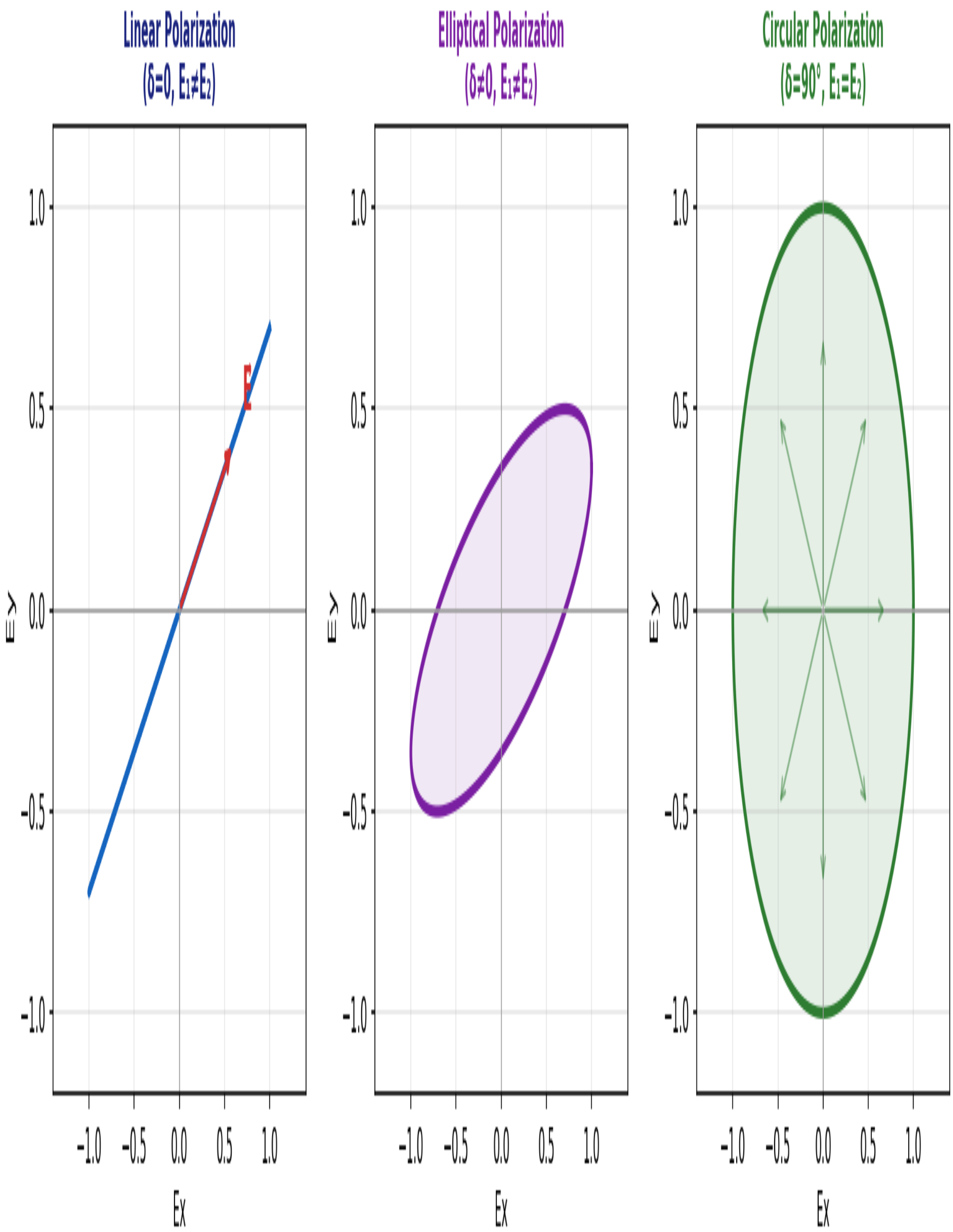

Figure 4.1: Three types of wave polarization

4.1 General Polarization Equation

The two components of E■ can be written as:

$$E_x = E■\cdot\cos(\omega t) \quad E_y = E■\cdot\cos(\omega t - \delta)$$

where δ is the phase difference between the two components. Eliminating ωt, the general polarization equation is:

$$(E_x/E■)^2 + (E_y/E■)^2 - 2(E_x/E■)(E_y/E■)\cos(\delta) = \sin^2(\delta)$$

Polarization Type	Condition	Locus of E■ tip	Equation
Linear	$\delta = 0$ (in phase)	Straight line through origin	$E_x/E■ = E_y/E■$ ($y = mx$)
Elliptical	$\delta \neq 0$, E■ ≠ E■	Ellipse	$(E_x/E■)^2 + (E_y/E■)^2 = 1$ (special)
Circular	$\delta = \pm 90°$, E■ = E■	Circle	$E_x^2 + E_y^2 = E■^2$

4.2 Conditions Summary

Condition 1 – Linear Polarization	If $\delta = 0$ (E_x and E_y in phase): the polarization equation reduces to $E_x/E■ = E_y/E■$, which is a straight line $y = mx$. The angle $\theta = \tan$■$^1(E_y/E_x)$ is constant with time.
Condition 2 – Elliptical Polarization	If $\delta \neq 0$ and E■ ≠ E■ (e.g. $\delta = \pi/2$): the equation becomes $(E_x/E■)^2 + (E_y/E■)^2 = 1$, an ellipse. E■ traces an ellipse as time progresses.
Condition 3 – Circular Polarization	If $\delta = \pm 90°$ AND E■ = E■: the equation reduces to $E_x^2 + E_y^2 = E■^2$, a circle. The tip of E■ rotates in a circle. This is the special case of elliptical polarization.

5. SOLVED EXAMPLES

Example 1 – Air Line (Lossless)

Problem: An air line has Z■ = 70 Ω and phase constant $\beta = 3$ rad/m at 100 MHz. Calculate L and C per metre of the line.

Solution:

Step 1: An air line is treated as a lossless line: $R = G = 0$, $\alpha = 0$.

Step 2: For lossless: Z■ = $\sqrt{L/C} = 70 \rightarrow L/C = 70^2 = 4900 \rightarrow L = 4900C$

Step 3: $\beta = \omega\sqrt{LC} = 3$, with $\omega = 2\pi\times100\times10$■ = 6.2832×10■ rad/s

Step 4: 6.2832×10■ × $\sqrt{4900C^2} = 3 \rightarrow$ 6.2832×10■ × 70C = 3

Step 5: C = 3/(6.2832×10■ × 70) = 6.82×10■^{11} F/m ≈ 68.2 pF/m

Step 6: L = 4900 × 68.2×10■^{12} = 334.2×10■■ H/m ≈ 334.2 nH/m

Example 2 – Distortionless Line

Problem: A distortionless line has $Z_\blacksquare = 60\ \Omega$, $\alpha = 20$ mNp/m, $u = 0.6c$. Find R, L, G, C and λ at 100 MHz.

Solution:

Step 1: For distortionless line: $R/L = G/C \rightarrow Z_\blacksquare = \sqrt{L/C} = \sqrt{R/G}$

Step 2: $\alpha = \sqrt{RG} = 20\times10^{\blacksquare 3} \rightarrow RG = (20\times10^{\blacksquare 3})^2 = 4\times10^{\blacksquare\blacksquare}$ … (1)

Step 3: $Z_\blacksquare = \sqrt{R/G} = 60 \rightarrow R/G = 3600 \rightarrow R = 3600G$ … (2)

Step 4: Sub (2) in (1): $3600G^2 = 4\times10^{\blacksquare\blacksquare} \rightarrow G = 3.33\times10^{\blacksquare\blacksquare}$ S/m

Step 5: $R = 3600 \times 3.33\times10^{\blacksquare\blacksquare} = 1.2\ \Omega$/m

Step 6: $u = 1/\sqrt{LC} = 0.6\times3\times10^{\blacksquare} = 1.8\times10^{\blacksquare}$ m/s $\rightarrow LC = 1/(1.8\times10^{\blacksquare})^2 = 3.086\times10^{\blacksquare 1\blacksquare}$

Step 7: $L/C = Z_\blacksquare^2 = 3600 \rightarrow C = \sqrt{3.086\times10^{\blacksquare 1\blacksquare}/3600} = 92.5$ pF/m

Step 8: $L = 3600 \times 92.5\times10^{\blacksquare 12} = 333\times10^{\blacksquare\blacksquare} = 333$ nH/m

Step 9: $\lambda = u/f = 1.8\times10^{\blacksquare}/10^{\blacksquare} = 1.8$ m

Example 3 – Terminated Lossless Line

Problem: A lossless TL with $Z_\blacksquare = 50\ \Omega$, 30 m long, operates at 2 MHz. Load $Z_L = 60+j40\ \Omega$, $u = 0.6c$. Find Γ, SWR, and Z_{in}.

Solution:

Step 1: $\Gamma_L = (Z_L - Z_\blacksquare)/(Z_L + Z_\blacksquare) = (60+j40-50)/(60+j40+50) = (10+j40)/(110+j40)$

Step 2: $|\Gamma_L| = |10+j40|/|110+j40| = \sqrt{100+1600}/\sqrt{12100+1600} = \sqrt{1700}/\sqrt{13700} = 0.3522$

Step 3: $VSWR = (1+|\Gamma|)/(1-|\Gamma|) = (1+0.3522)/(1-0.3522) = 1.3522/0.6478 = 2.087$

Step 4: $\beta = \omega/u = 2\pi\times2\times10^{\blacksquare}/(0.6\times3\times10^{\blacksquare}) = 0.06981$ rad/m

Step 5: $\beta l = 0.06981 \times 30 = 2.094$ rad $= 2\pi/3 \rightarrow \tan(\beta l) = \tan(120°) = -\sqrt{3}$

Step 6: $Z_{in} = 50 \times [60+j40 + j50\times(-\sqrt{3})] / [50 + j(60+j40)\times(-\sqrt{3})]$

Step 7: $Z_{in} \approx 23.97 + j1.35\ \Omega$

End of Module IV Notes – Electromagnetic Theory