ELECTROMAGNETIC WAVES

MODULE 3 — Digital Notes

Reflection & Refraction · Wave Propagation · Polarization

- Poynting Vector & Theorem
- Normal Incidence: Reflection & Transmission
- Oblique Incidence: Parallel & Perpendicular Polarization
- Wave Propagation in Lossy Dielectric, Free Space, Good Conductors
- Attenuation, Phase Constants & Skin Depth
- Polarization of Plane Waves (Linear, Elliptical, Circular)
- Maxwell's Equations (Phasor Form) · Brewster Angle

1. Poynting Vector & Poynting's Theorem

Poynting Vector (S■) — denoted **S■ = E■ × H■** — measures the *rate of energy flow* per unit area of an EM wave. Its direction is the direction of power flow and it is perpendicular to the plane containing E■ and H■.

S■ = E■ × H■	**|S■| = |E■||H■| sin(E■, H■)**
Units: W/m²	**P■ = E■ × H■ (Poynting Vector)**

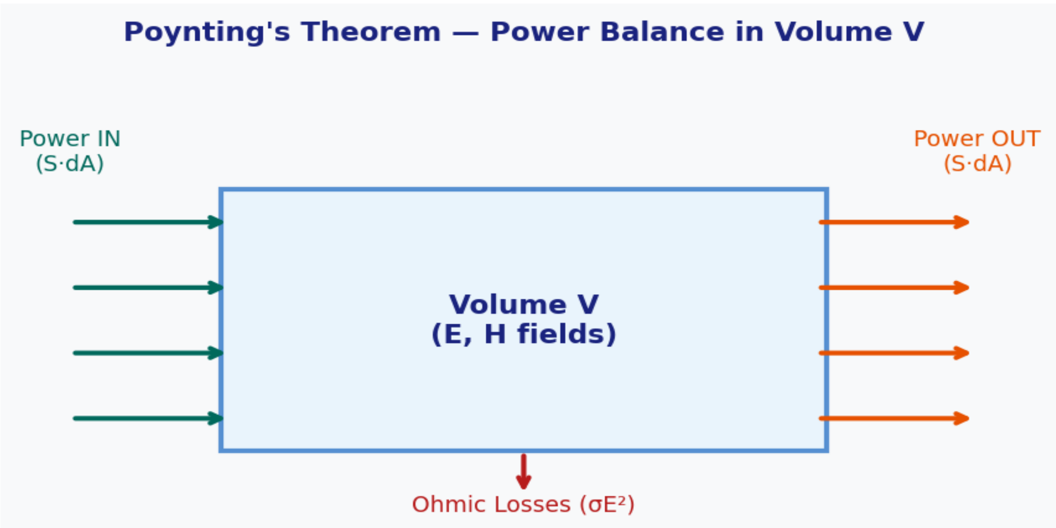
Poynting's Theorem (Statement)

The vector product of electric field intensity and magnetic field intensity at any point is a measure of the rate of energy flow per unit area at that point.

Net power flowing out of a volume V = Rate of decrease in stored energy (electric + magnetic) within V, minus conduction (ohmic) losses.

Mathematical Form (from Maxwell's equations):

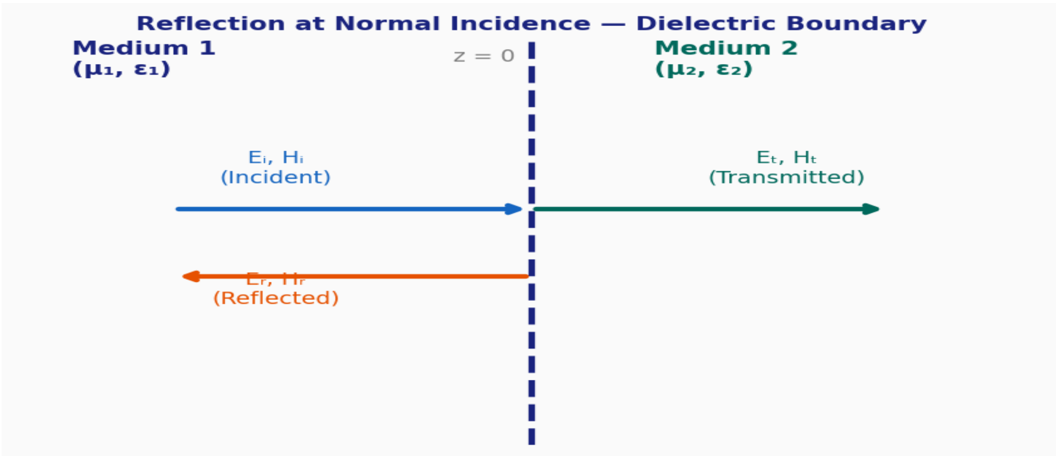
■ (E■×H■)·dS = –d/dt ∫[½εE² + ½μH²]dV – ∫σE²dV
Total power out = –d(W■+W■)/dt – Ohmic losses



2. Reflection & Refraction — Normal Incidence

Consider a plane wave in **Medium 1 (μ■, ε■)** striking the boundary z = 0 with **Medium 2 (μ■, ε■)**. The intrinsic impedances are:

η■ = √(μ■/ε■)	**η■ = √(μ■/ε■)**
E■/H■ = η■	**E■/H■ = η■**



Boundary Conditions (tangential components continuous):

E■ = E■ + E■ and H■ = H■ + H■

Reflection Coefficient (Γ)

Γ = E■/E■ = (η■ – η■) / (η■ + η■)
For lossless dielectrics: Γ = (√ε■ – √ε■) / (√ε■ + √ε■)

Transmission Coefficient (τ)

τ = E■/E■ = 2η■ / (η■ + η■)
For lossless dielectrics: τ = 2√ε■ / (√ε■ + √ε■)
Relation: τ = 1 + Γ

Magnetic Field Coefficients

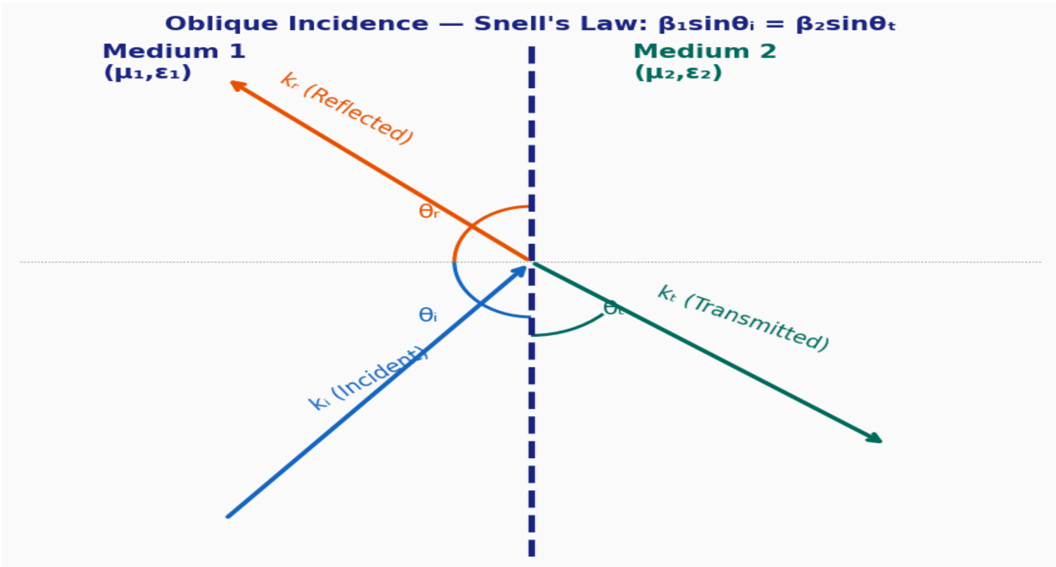
H■/H■ = (η■ – η■)/(η■ + η■)	**H■/H■ = 2η■/(η■ + η■)**

3. Oblique Incidence

When a wave strikes a boundary at an angle, it obeys **Snell's Law**. There are two polarization cases:

Snell's Law of Refraction:

β■ sin θ■ = β■ sin θ■   (and θ■ = θ■ — Law of Reflection)



3a. Parallel Polarization (TM / p-pol)

Electric field E■ lies *in the plane of incidence* (the x-z plane).

Γ■ = E■■/E■■ = (η■cosθ■ – η■cosθ■) / (η■cosθ■ + η■cosθ■)
τ■ = E■■/E■■ = 2η■cosθ■ / (η■cosθ■ + η■cosθ■)

3b. Perpendicular Polarization (TE / s-pol)

Electric field E■ is *perpendicular to* the plane of incidence; magnetic field H■ is *parallel* to the plane of incidence.

Γ⊥ = E■■/E■■ = (η■cosθ■ – η■cosθ■) / (η■cosθ■ + η■cosθ■)
τ⊥ = E■■/E■■ = 2η■cosθ■ / (η■cosθ■ + η■cosθ■)
Relation: 1 + Γ⊥ = τ⊥

4. EM Wave Propagation — Different Media

Medium	σ	ε	α	Key Property
Free Space	σ = 0	ε = ε■	0	No attenuation
Lossless Dielectric	σ = 0	ε = ε■ε■	0	E, H in phase
Lossy Dielectric	σ ≠ 0	ε = ε■ε■	α > 0	Complex η
Good Conductor	σ >> ωε	ε = ε■ε■	= β = √(πfμσ)	E leads H by 45°

Vector Helmholtz Equation

∇²E■ – γ²E■ = 0 and ∇²H■ – γ²H■ = 0
where γ² = jωμ(σ + jωε) ■ γ = α + jβ

Propagation Constant γ = α + jβ

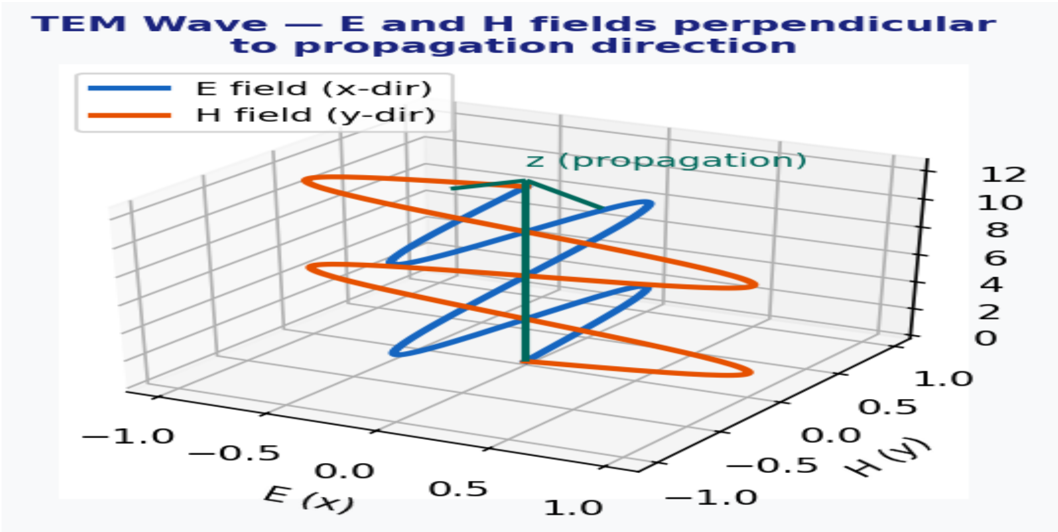
Constant	Name	Units	Physical Meaning
α	Attenuation constant	Np/m	Amplitude decay per unit length
β	Phase-shift constant	rad/m	Phase change per unit length

General Expressions for α and β:

α = ω√(με/2) · [√(1+(σ/ωε)²) – 1]^½
β = ω√(με/2) · [√(1+(σ/ωε)²) + 1]^½
Phase Velocity: V■ = ω/β = λf = 1/√(με)

4a. Wave Propagation in Free Space (σ = 0, ε = ε■, μ = μ■)

α = 0	**β = ω√(μ■ε■) = ω/c**
η = √(μ■/ε■) = 120π ≈ 377 Ω	**θ■ = 0° (E and H in phase)**
E■ = E■ cos(ωt – βz) â■	**H■ = (E■/η) cos(ωt – βz) â_y**


TEM Wave — E and H fields perpendicular to propagation direction
E field (x-dir)
H field (y-dir)
z (propagation)
12
10
8
6
4
2
0
−1.0
−0.5
0.0
0.5
1.0
E (x)
1.0
0.5
0.0
−0.5
−1.0
H (y)

4b. Wave Propagation in Lossless (Perfect) Dielectric (σ = 0)

α = 0	β = ω√(με)
V■ = 1/√(με)	η = √(μ/ε) ∠0°
E■ = E■cos(ωt–βz)â■	H■ = (E■/η)cos(ωt–βz)â_y

Loss Tangent: *tan θ = σ/ωε (ratio of conduction current to displacement current)*

4c. Wave Propagation in Good Conductors (σ >> ωε)

α = β = √(ωμσ/2) = √(πfμσ)	
η = √(jωμ/σ) = √(ωμ/σ) ∠45°	
E■ leads H■ by 45°	
E■ = E■e■■■cos(ωt–βz)â■	H■ = (E■/|η|)e■■■cos(ωt–βz–45°)â_y

4d. Intrinsic Impedance η

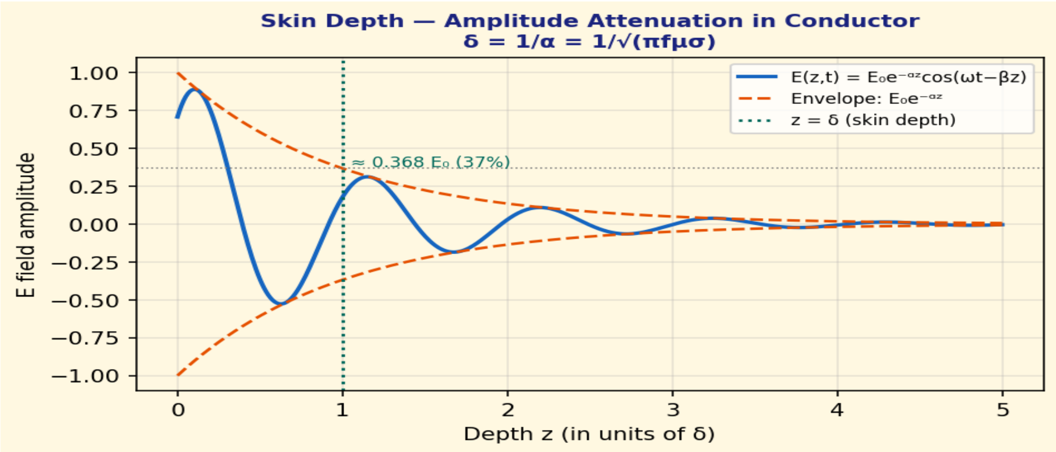
η = √(jωμ / (σ + jωε))
tan 2θ■ = σ/ωε (phase angle between E and H)

5. Skin Depth (Depth of Penetration)

In a conducting medium a wave gets progressively attenuated. **Skin depth** δ is the distance through which the wave amplitude decreases by a factor e^{-1} (≈ 37%) of its original value.

$$\delta = 1/\alpha = 1/\sqrt{\pi f \mu \sigma}$$

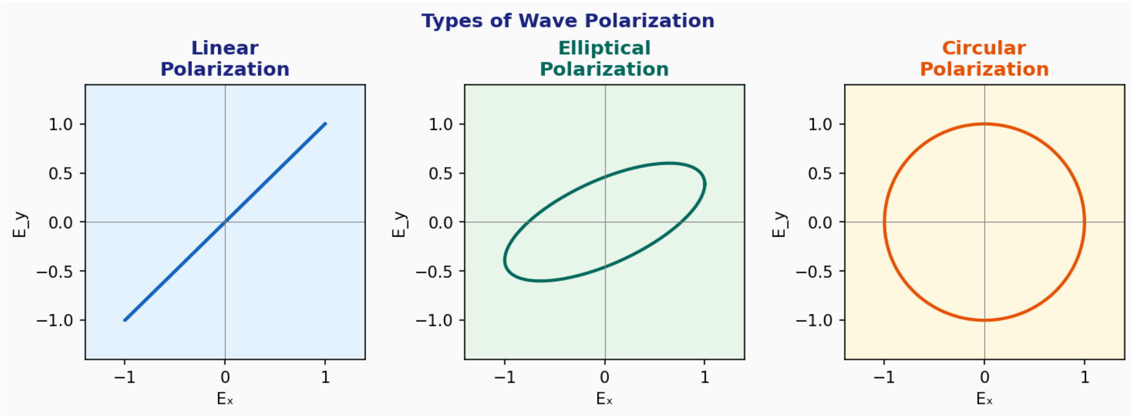
At $z = \delta$: $E = E_0 e^{-1} \approx 0.368 E_0$



Skin Effect: At high frequencies charges migrate to the surface of a conductor. Fields and currents are confined to a thin skin layer of thickness δ. This is why hollow tubular conductors are used instead of solid conductors in outdoor TV antennas.

6. Polarization of Plane Waves

The **polarization** of a uniform plane wave is defined as the time-varying behaviour of the electric field vector E■ at a fixed point in space along the direction of propagation.



General Polarization Equation (sinusoidal wave):

$$(E_\blacksquare/E_\blacksquare)^2 + (E_y/E_\blacksquare)^2 - 2(E_\blacksquare/E_\blacksquare)(E_y/E_\blacksquare)\cos\delta = \sin^2\delta$$

where δ = phase difference between the two components

Type	Condition	Result
Linear	$\delta = 0$ (or $n\pi$)	Straight line: $E_\blacksquare = (E_\blacksquare/E_\blacksquare)E_y$
Elliptical	$E_\blacksquare \neq E_\blacksquare$, $\delta \neq 0, \pi/2$	Ellipse: $(E_\blacksquare/E_\blacksquare)^2 + (E_y/E_\blacksquare)^2 = 1$ (general)
Circular	$E_\blacksquare = E_\blacksquare$, $\delta = \pi/2$	Circle: $E_\blacksquare^2 + E_y^2 = 1$

7. Maxwell's Equations — Phasor (Time-Varying) Form

Since most generators produce sinusoidally varying voltages and currents, we replace $\partial/\partial t$ by $j\omega$ in Maxwell's equations:

Equation	Time Domain	Phasor Form
Faraday's Law	$\nabla\times E\blacksquare = -\mu\ \partial H\blacksquare/\partial t$	$\nabla\times E\blacksquare = -j\omega\mu H\blacksquare$
Ampere's Law	$\nabla\times H\blacksquare = \sigma E\blacksquare + \varepsilon\partial E\blacksquare/\partial t$	$\nabla\times H\blacksquare = (\sigma + j\omega\varepsilon)E\blacksquare$
Gauss (Electric)	$\nabla\cdot D\blacksquare = \rho\blacksquare$	$\nabla\cdot D\blacksquare = \rho\blacksquare$
Gauss (Magnetic)	$\nabla\cdot B\blacksquare = 0$	$\nabla\cdot B\blacksquare = 0$

Wave Equations in Free Space:

$\nabla^2 E\blacksquare = \mu\blacksquare\varepsilon\blacksquare\ \partial^2 E\blacksquare/\partial t^2 = (1/u^2)\ \partial^2 E\blacksquare/\partial t^2$
$\nabla^2 H\blacksquare = (1/u^2)\ \partial^2 H\blacksquare/\partial t^2$
$u = 1/\sqrt{(\mu\blacksquare\varepsilon\blacksquare)} = c = 3\times 10\blacksquare$ m/s **(speed of light)**

8. Brewster Angle (θ_Bi)

The **Brewster angle** (also called the *polarizing angle*) is the angle of incidence at which the **reflection coefficient for parallel polarization becomes zero** (Γ■ = 0). At this angle, the reflected wave is completely absent for the parallel component.

Condition: η■cosθ■ = η■cosθ_Bi ■ Γ■ = 0
For lossless dielectrics (μ■ = μ■ = μ■):
sin θ_Bi = √(ε■/(ε■+ε■))
tan θ_Bi = √(ε■/ε■) = n■/n■

Key Point: The Brewster angle exists for **any** combination of ε■ and ε■. At θ_Bi, all of the reflected light (for parallel polarization) vanishes — used in Polaroid sunglasses and anti-reflection coatings.

Quick Reference — Key Formulas

Formula	Expression	Notes
Poynting Vector	S■ = E■ × H■	W/m²
Reflection Coeff.	Γ = (η■–η■)/(η■+η■)	Normal incidence
Transmission Coeff.	τ = 2η■/(η■+η■)	1+Γ = τ
Propagation const.	γ = α + jβ	γ² = jωμ(σ+jωε)
Attenuation const.	α = ω√(με/2)[√(1+(σ/ωε)²)–1]^½	Np/m
Phase const.	β = ω√(με/2)[√(1+(σ/ωε)²)+1]^½	rad/m
Phase velocity	V■ = ω/β = 1/√(με)	m/s
Intrinsic impedance	η = √(jωμ/(σ+jωε))	Ohms
Skin depth	δ = 1/α = 1/√(πfμσ)	Good conductor
Brewster angle	tan θ_Bi = √(ε■/ε■)	μ■ = μ■ = μ■
Free space η	η■ = √(μ■/ε■) = 120π ≈ 377Ω	Intrinsic impedance
Snell's Law	β■sinθ■ = β■sinθ■	Oblique incidence