ELECTROMAGNETICS THEORY · MODULE 1

Intro to Electromagnetics

Vector Calculus · Coordinate Systems · Electrostatics · Maxwell's Equations

CONTENTS

01 Scalars & Vectors

SCALAR

A quantity with **magnitude only**.

e.g. time, mass, distance, temperature, electric potential

VECTOR

A quantity with **magnitude and direction**.

e.g. velocity, force, displacement, electric field intensity

Unit Vector

A vector of magnitude unity whose direction is along A:

DEFINITION

$$\hat{a}_A = \mathbf{A} / |\mathbf{A}| \implies \mathbf{A} = |A|\,\hat{a}_A$$

In Cartesian coordinates:

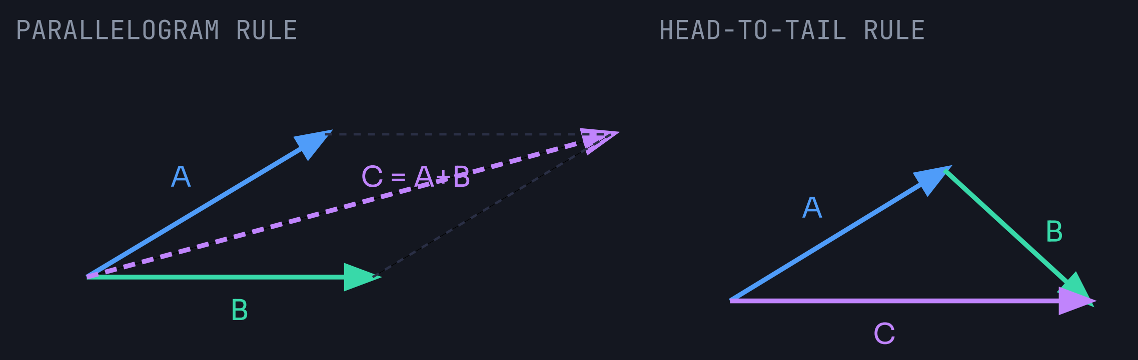
CARTESIAN REPRESENTATION

$$\mathbf{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$|\mathbf{A}| = \sqrt{(A_x^2 + A_y^2 + A_z^2)}$$

Vector Addition & Subtraction

PARALLELOGRAM & HEAD-TO-TAIL RULES



IN COMPONENT FORM

$$\mathbf{A} + \mathbf{B} = (A_x+B_x)\hat{a}_x + (A_y+B_y)\hat{a}_y + (A_z+B_z)\hat{a}_z$$

$$\mathbf{A} - \mathbf{B} = (A_x-B_x)\hat{a}_x + (A_y-B_y)\hat{a}_y + (A_z-B_z)\hat{a}_z$$

02 Vector Products

DOT PRODUCT (SCALAR)

CROSS PRODUCT (VECTOR)

$$\mathbf{A}\cdot\mathbf{B} = |A||B| \cos\theta$$
$$= A_xB_x + A_yB_y + A_zB_z$$

If **A**·**B** = 0 → vectors are *orthogonal*

$$\mathbf{A}\times\mathbf{B} = |A||B| \sin\theta\ \hat{a}_n$$

If **A**×**B** = 0 → vectors are *parallel*

Direction: right-hand rule from A to B

Cross Product – Determinant Form

3×3 DETERMINANT EXPANSION

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{a}_x - (A_xB_z - A_zB_x)\hat{a}_y + (A_xB_y - A_yB_x)\hat{a}_z$$

Special Products

SCALAR TRIPLE PRODUCT

$$A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$$

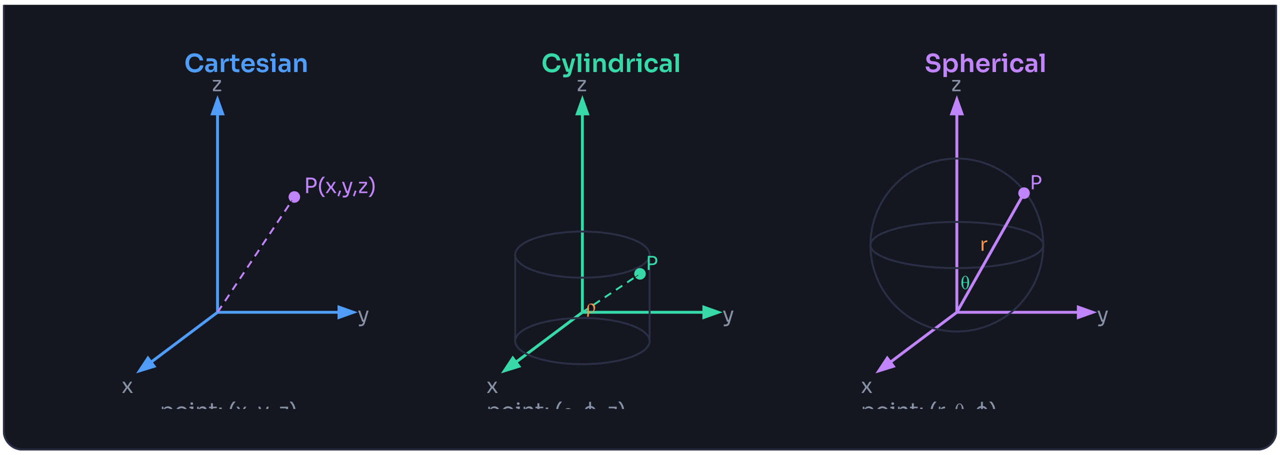
VECTOR TRIPLE PRODUCT

$$A\times(B\times C) = B(A\cdot C) - C(A\cdot B)$$

03

Coordinate Systems

THREE MAJOR COORDINATE SYSTEMS



Coordinate Transformations

FROM → TO	KEY RELATIONS
Cartesian → Cylindrical	$\rho = \sqrt{(x^2+y^2)}$, $\phi = \tan^{-1}(y/x)$, $z = z$
Cylindrical → Cartesian	$x = \rho \cos\phi$, $y = \rho \sin\phi$, $z = z$
Cartesian → Spherical	$r = \sqrt{(x^2+y^2+z^2)}$, $\theta = \tan^{-1}(\sqrt{(x^2+y^2)}/z)$, $\phi = \tan^{-1}(y/x)$
Spherical → Cartesian	$x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$
Cylindrical → Spherical	$r = \sqrt{(\rho^2+z^2)}$, $\theta = \tan^{-1}(\rho/z)$, $\phi = \phi$
Spherical → Cylindrical	$\rho = r \sin\theta$, $\phi = \phi$, $z = r \cos\theta$

Differential Elements

CARTESIAN

$d\mathbf{l} = dx\ \hat{a}_x + dy\ \hat{a}_y + dz\ \hat{a}_z$

$dV = dx\ dy\ dz$

CYLINDRICAL

$d\mathbf{l} = d\rho\ \hat{a}_\rho + \rho d\phi\ \hat{a}_\phi + dz\ \hat{a}_z$

$dV = \rho\ d\phi\ d\rho\ dz$

SPHERICAL

$d\mathbf{l} = dr\ \hat{a}_r + r\ d\theta\ \hat{a}_\theta + r\ \sin\theta\ d\phi\ \hat{a}_\phi$

$dV = r^2\ \sin\theta\ dr\ d\theta\ d\phi$

04 Del Operator (∇)

DEFINITION IN COORDINATE SYSTEMS

CARTESIAN

$$\nabla = (\partial/\partial x)\ \hat{a}_x + (\partial/\partial y)\ \hat{a}_y + (\partial/\partial z)\ \hat{a}_z$$

CYLINDRICAL

$$\nabla = (\partial/\partial \rho)\ \hat{a}_\rho + (1/\rho)(\partial/\partial \phi)\ \hat{a}_\phi + (\partial/\partial z)\ \hat{a}_z$$

SPHERICAL

$$\nabla = (\partial/\partial r)\ \hat{a}_r + (1/r)(\partial/\partial \theta)\ \hat{a}_\theta + (1/r \sin\theta)(\partial/\partial \phi)\ \hat{a}_\phi$$

FOUR OPERATIONS WITH ∇

Divergence

The divergence of vector **A** at a point P is the outward flux per unit volume as the volume shrinks about P. It measures how much the field diverges from that point.

CARTESIAN

$$\nabla\cdot\mathbf{A} = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$$

CYLINDRICAL

$$\nabla\cdot\mathbf{A} = (1/\rho)\ \partial(\rho A_\rho)/\partial \rho + (1/\rho)\ \partial A_\phi/\partial \phi + \partial A_z/\partial z$$

KEY TERMS

- **Solenoidal vector**: divergence = 0 (no net outflow at any point)
- **Divergence theorem**: $\oint_S A \cdot ds = \iiint_V \nabla \cdot A \, dv$

Curl

The curl of a vector measures the tendency of the vector to rotate or twist. It is defined as the maximum circulation per unit area as the area tends to zero.

CARTESIAN (DETERMINANT FORM)

$$\nabla \times \mathbf{A} = |\hat{a}_x \ \hat{a}_y \ \hat{a}_z; \ \partial/\partial x \ \partial/\partial y \ \partial/\partial z; \ A_x \ A_y \ A_z|$$

KEY TERMS

- **Irrotational vector**: curl = 0
- **Stokes' theorem**: $\oint_L A \cdot dl = \iint_S (\nabla \times A) \cdot ds$

Gradient

The gradient of any scalar is the maximum space rate of change of that function. It results in a vector.

CARTESIAN

$$\nabla V = (\partial V/\partial x) \ \hat{a}_x + (\partial V/\partial y) \ \hat{a}_y + (\partial V/\partial z) \ \hat{a}_z$$

05 Electrostatics & Coulomb's Law

Electrostatics is the study of phenomena associated with static charges or electricity at rest.

Coulomb's Law

The force between two point charges Q_1 and Q_2 is:

- Along the line joining them
- Directly proportional to the product Q_1Q_2
- Inversely proportional to the square of the distance R between them

VECTOR FORM

$$F_{12} = Q_1Q_2\,\hat{R}_{12} / (4\pi\varepsilon_0 R^2)$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m (permittivity of free space)

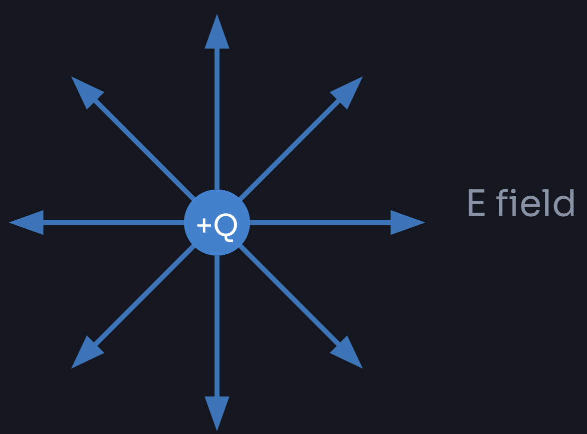
$k = 1/(4\pi\varepsilon_0) \approx 9\times10^9$ m/F

$R_{12} = r_2 - r_1,\quad R = |R_{12}|$

Electric Field Intensity (E)

The electric field intensity E is the force per unit charge. It is the force that a small positive test charge would experience.

ELECTRIC FIELD LINES FROM A POINT CHARGE



Field radiates outward from positive charge

ELECTRIC FIELD DUE TO POINT CHARGE Q

$$E = Q\,\hat{a}_R / (4\pi\varepsilon_0 R^2) \quad N/C$$

For N charges: $E = (1/4\pi\varepsilon_0)\ \Sigma\ Q_k(r - r_k) / |r - r_k|^3$

Electric Flux Density (D)

RELATION BETWEEN D AND E

$$D = \varepsilon_0 E = Q\,\hat{a}_R / (4\pi R^2)$$

Unit of D: C/m^2 | In any medium: $D = \varepsilon E$, $\varepsilon = \varepsilon_0\varepsilon_r$

Charge Distributions

LINE CHARGE (ρ_L)

$dQ = \rho_L\, dl$

$Q = \int \rho_L\, dl$

Unit: C/m

SURFACE CHARGE (ρ_S)

$dQ = \rho_S\, ds$

$Q = \iint \rho_S\, ds$

Unit: C/m^2

VOLUME CHARGE (ρ_V)

$dQ = \rho_V\, dv$

$Q = \iiint \rho_V\, dv$

Unit: C/m^3

06 Gauss's Law

Statement: The total electric flux ψ through any closed surface is equal to the total charge Q enclosed by that surface. $\psi = Q_{enc}$

INTEGRAL FORM

$$\oiint_S D\cdot ds = Q_{enc} = \iiint_V \rho_V\, dv$$

DIFFERENTIAL/POINT FORM — MAXWELL'S 1ST EQUATION

$$\nabla \cdot D = \rho_V$$

Applications: Gaussian Surface Method

POINT CHARGE

Choose *spherical* Gaussian surface.

$$E = Q\, \hat{a}_r / (4\pi\varepsilon o r^2)$$

INFINITE LINE CHARGE

Choose *cylindrical* Gaussian surface.

$$E = \rho_L\, \hat{a}_\rho / (2\pi\varepsilon o \rho)$$

07 Electric Potential (V)

The potential difference V_{AB} is the work done per unit charge in moving a positive test charge from A to B against the electric field.

POTENTIAL DIFFERENCE

$$V_{AB} = W/Q = -\int_A^B E \cdot dl$$

KEY RELATIONSHIPS

$E = -\nabla V$ (Electric field is negative gradient of potential)

- $\oint E \cdot dl = 0$ (Conservative field — no work done around a closed path)
- $\nabla \times E = 0$ (Maxwell's 2nd equation for static fields)

Energy in Electrostatic Fields

FOR N POINT CHARGES

$$W_E = (1/2)\ \Sigma\ Q_k V_k$$

ENERGY DENSITY — VOLUME DISTRIBUTION

$$W_E = (\varepsilon/2) \int E^2\, dv = (1/2) \int D \cdot E\, dv$$

08 Poisson's & Laplace Equations

DERIVATION

From Gauss's law: $\nabla \cdot D = \rho_V$, and $D = \varepsilon E$, and $E = -\nabla V$:

$\nabla \cdot \varepsilon(-\nabla V) = \rho_V \implies \nabla \cdot \nabla V = -\rho_V/\varepsilon$

POISSON'S EQUATION

$$\nabla^2 V = -\rho_V/\varepsilon$$

Where $\rho_V \neq 0$

LAPLACE'S EQUATION

$$\nabla^2 V = 0$$

Charge-free region: $\rho_V = 0$

Laplace's Equation in Coordinate Systems

SYSTEM	LAPLACE'S EQUATION $\nabla^2 V = 0$
Cartesian	$\partial^2V/\partial x^2 + \partial^2V/\partial y^2 + \partial^2V/\partial z^2 = 0$
Cylindrical	$(1/\rho)\ \partial/\partial\rho(\rho\ \partial V/\partial\rho) + (1/\rho^2)\ \partial^2V/\partial\phi^2 + \partial^2V/\partial z^2 = 0$
Spherical	$(1/r^2)\ \partial/\partial r(r^2\ \partial V/\partial r) + (1/r^2 \sin\theta)\ \partial/\partial\theta(\sin\theta\ \partial V/\partial\theta) + (1/r^2 \sin^2\theta)\ \partial^2V/\partial\phi^2 = 0$

09 Ampere's Circuit Law

Statement: The line integral of H (magnetic field intensity) around a closed path equals the net current enclosed by the path.

$\oint H\cdot dl = I_{enc}$

Applying Stokes' theorem: $\oint H\cdot dl = \iint_S (\nabla\times H)\cdot ds$. Since $I_{enc} = \iint_S J\cdot ds$:

POINT FORM — MAXWELL'S 3RD EQUATION (STATIC)

$$\nabla\times H = J$$

Magnetic Flux Density (B)

RELATIONSHIP TO H

$B = \mu_0 H$ (in free space)

$\mu_0 = 4\pi\times10^{-7}$ H/m (permeability of free space)

MAGNETIC FLUX THROUGH SURFACE S

$\psi_m = \int_S B\cdot ds$

KEY PROPERTY

Magnetic flux lines always close upon themselves — isolated magnetic poles do not exist. Therefore:

$$\oiint_S B\cdot ds = 0 \implies \nabla\cdot B = 0 \quad \text{(Maxwell's 4th equation)}$$

10 Maxwell's Equations (Statics)

1ST EQUATION (GAUSS — ELECTRIC)

$$\nabla \cdot D = \rho_V$$

Electric flux diverges from charge

2ND EQUATION (FARADAY — STATIC)

$$\nabla \times E = 0$$

Electrostatic field is conservative

3RD EQUATION (AMPERE — STATIC)

$$\nabla \times H = J$$

Magnetic field circulates around current

4TH EQUATION (GAUSS — MAGNETIC)

$$\nabla \cdot B = 0$$

No isolated magnetic poles

EQUATION	INTEGRAL FORM	DIFFERENTIAL FORM
Gauss (E)	$\oiint D \cdot ds = Q_{enc}$	$\nabla \cdot D = \rho_V$
Faraday (static)	$\oint E \cdot dl = 0$	$\nabla \times E = 0$
Ampere (static)	$\oint H \cdot dl = I_{enc}$	$\nabla \times H = J$
Gauss (B)	$\oiint B \cdot ds = 0$	$\nabla \cdot B = 0$

Hello World!